Rationality and Quality of Inflation Forecasts

Victoria Petrenko

Université Paris1

UFR 02 Sciences Economiques

Mention du Master – Master 2 Recherche ETE

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Nom du directeur de la soutenance: Terracol Antoine

Présenté et soutenu par: Petrenko Victoria

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Introduction

Most of existing methods of forecasting economic variables include evaluation of forecast's accuracy relative to competing models. The most widespread method includes calculation of mean absolute forecast error, mean absolute percentage error or root mean squared prediction error. After that the choice of the forecasting model is done on the basis of average errors comparison. For this purpose a wide range of exact, asymptotic or non-parametric tests might be used for calculation significance of the difference in models' behavior. This paper focuses on another measure of forecast's quality: rationality.

The theory of rational expectations originally proposed by John Muth and further developed by Robert Lucas is one of the most powerful macroeconomic concepts used nowadays. The importance of this concept is justified by the fact that Lucas was awarded Nobel Memorial Prize in 1995 "for having developed and applied the hypothesis of rational expectations, and thereby having transformed macroeconomic analysis and deepened our understanding of economic policy"[1].

Rational expectations are used in the variety of economic models for both scientific and applied work. The most outstanding example is Dynamic Stochastic General Equilibrium models. Not only macroeconomic researchers take advantage of DSGE models: these models are exploited by the majority of Central Banks including Federal Reserve, Bank of England and European Central Bank.

Among the advantages of the models with rational expectations are solid theoretical foundations, developed tools for their solution and their capability of producing macroeconomic variables' dynamics with smoothed trajectories. However, the assumption that the agent uses all available information is a very rigid condition. In practice the acquisition of the information is associated with high costs. Moreover, only a part of information might be enough for an agent to make economic decision.

In this paper rationality of the forecaster is treated as minimization of her loss function taking into consideration all available information. We investigate whether the forecasters make their projections in accordance with proposed notion of rationality. Experts whose projections are reported in Survey of Professional Forecasters represent large banks, thinktanks, investment banks, consultancy and large corporations. Their forecasts are important because market participants use this information to form their own expectations about economic variables. Moreover, professional forecasters have sufficient knowledge in economics as well as ample possibilities for data collection and processing. This makes forecasters perfect candidates for rationality check. The rejection of the rational expectation hypothesis for these agents would be a reason to doubt validity of the hypothesis.

Many applied econometric papers (for instance, Ang, Bekaert and Wei (2007) show that consensus forecasts outperform other econometric models in terms of prediction error. In this study we analyze a question whether rationality criterion could be used for obtaining more accurate forecast than simple mean or median of individual forecasts. It is quite intuitive to assume that rational forecasters make more accurate projections than non-rational ones because of taking into consideration all available information. If so, averaging of only rational forecasts might result in more accurate consensus forecast than average over all forecasters.

Finally, rationality tests were conducted for mean and median forecasts. Rationality of aggregate forecast would justify the idea that not all of agents might be rational on the individual level but their aggregate behavior might be rational. In addition to that, consensus forecasts are more important for economic agents in practice than individual ones. The reason for this is that aggregate forecasts are

[1] "the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1995

always published by organizations that conduct surveys whereas individual data is sometimes not available for the public.

Next sections are organized as follows: Section 1 summarizes existing literature on the topic of forecast's rationality. Section 2 contains details about theoretical aspects of econometric rationality testing. Section 3 describes two datasets used for the study. Section 4 covers econometric issues and Section 5 summarizes results.

Section 1. Literature review

In economic literature rationality is usually associated with the ability of the agent to maximize (or minimize) her objective function using expectation for the future values of variables. This approach is dominant in the macroeconomic theory at the moment. However, in spite of its solid theoretical background it demands strict assumptions about the information set available to the agent. According to the rational expectations hypothesis the agent should have some information about future realization of the random variable: for example, distribution function, density function or some probabilistic beliefs. These assumptions are much stricter than assumptions necessary for adaptive expectations for which only information about current and past values of the random variable are needed. The calculation of the future expectation of a random variable is even easier for naïve expectations when an agent believes that realization of the random variable in the future will be exactly the same as in the present moment.

In econometric literature concerning forecast rationality evaluation it is explicitly or implicitly assumed that the forecaster minimizes loss function. In other words, the deviation of the projection made by the forecaster from the actual value of the economic variable results in forecaster's disutility.

The most commonly used type of loss function is symmetric squared loss function according to which the forecaster minimizes the expectation of the squared forecast error. However, it is not difficult to imagine situations when positive and negative errors might involve different loss. That is why more recent literature concerning forecast rationality takes into consideration asymmetric loss functions. According to asymmetric loss functions positive and negative forecast errors enter the loss function with different weights.

In the paper Muth (1961) the author determined rationality as the fact that expectations of the firms are distributed about the objective probability distribution of outcomes. Or equivalently predictions are generated by the same stochastic process as the variable to be forecasted. However, it was tested in the early works of forecast rationality as whether expectations take into account all the information in the information set available at the moment of producing the forecast.

In Pesando (1975) Muth's type of rationality was tested as whether expectations incorporate all the past information about inflation dynamic. For this purpose Livingston's database was used. It contained CPI forecasts 6 and 12 month ahead. All available observations (first quarter 1959 – second quarter 1962) were divided into two intersecting subsamples: first quarter 1959 – first quarter 1969 and first quarter 1962 – second quarter 1969 Then using OLS three equations were estimated: actual CPI autoregression and two ADL models with projection as dependent variable and lags of actual CPI as exogenous variables. The hypothesis of rationality was implemented as Chow test that coefficient in all three equations are the same. Null hypothesis was rejected for both subsamples.

Carlson (1977) used the same method for rationality tests as Pesandro. The main difference of his article consisted in a more careful work with Livingston's data. In particular, the dataset was a subject to Livingston's revision for the purpose of publishing. The problem consisted in different timing of participants' answers. When abrupt changes in inflation dynamics occurred, forecasters who sent their projections later (and therefore presumably were able to see it from the published data) did better forecasts than participants who sent their results before data release. This was the reason why Livingston revised the dataset before publishing.

Carlson argued that revisions made by Livingston might have an impact on the rejection of null hypothesis in the previous paper. For this reason he advocated the use of unrevised version of data. However, it did not influence rationality results: rationality hypothesis was also rejected.

Mullineaux (1978) argued that Carlson and Pesando used wrong rationality test. Chow test assumes that error term is identically distributed in all three autoregressive equations, which might not be the case. He used Bartlett's statistic to test whether the variance of the series are the same. Due to rejection of the null hypothesis for both datasets he concluded that Chow test previously used was inappropriate. As a consequence, alternative rationality test was proposed. He separately estimated two OLS equations. First equation was a regression of 6 month forecast error on the lags of CPI. This equation is equivalent to the difference of the first and second equations discussed before.

The second equation was obtained as a difference of second and third equation considered above. Rationality hypothesis was checked as equality of all the coefficients to zero in two equations separately. As a consequence, modified rationality hypothesis was not rejected for both datasets.

Mishkin (1981) examined rationality of interested rates and inflation. For this purpose he used the survey of current business. He conducted the test similar to Mullineaux (1978). Mishkin used an OLS regression of the forecast error on the lags of inflation. Afterwards rationality hypothesis was tested as simultaneous equality to zero of all coefficients in the equation. The rationality hypothesis was rejected for the period first quarter 1959 – end 1969. However, it was not rejected for subsample first quarter 1954 – end 1976. The author explained this phenomenon by the specific of the first period. In the beginning of the first subsample inflation was low and then it was rising to the higher level.

Keane, Runkle (1989) tested GDP deflator for sixteen professional forecasters. The authors used ASA-NBER database from last quarter 1968 till third quarter 1986. In the article different models were estimated where were present different combinations of unbiasness of the forecasts, absence of autocorrelation of the forecast error and absence of correlation with different economic variables. Null hypothesis about rationality was rejected for most of the models except for one. In this model current price level is included which is not available for the forecaster because of delays in data publishing. The main result of this article, according to the authors, is that it is important to use only projections produced by professional forecasters.

In Keane, Runkle (1990) authors used the same dataset. They succeeded to show that the forecast are rational contradicting their previous results. This result was achieved by the use of panel data instead of aggregated or average forecast. Moreover, they used only forecasts produced by professional forecasters. From the authors' point of view, the reason for this is that experts bear reputational risk when they make a projection, unlike other participants. Consequently, professional forecasters take more reasonable decisions and thoughtfully analyze all available information. Authors believe that this behavior of professional forecasters is the reason why they satisfy theoretical outline.

Bonham, Dacy (1991) considered several models for inflation forecasting: model using interest rates, Phillips curve, three variants of time series models, average of the above-described models and, finally, consensus ASA-NBER forecast. Time span from 1970 till the middle of 1984 was used.

The authors used three definitions of the rationality:

- weak rationality, which is equivalent to the forecast's unbiasness;
- sufficient rationality, which states the absence of the autocorrelation of the forecast error;
- strong rationality, according to which conditions necessary for weak and sufficient rationality must be satisfied for any period.

The authors' research showed that none of the forecasting methods satisfies the definition of the strong rationality. As a consequence, conclusion is made that rationality is a very demanding assumption.

Aggarwal, Mohanty, Song (1995) used symmetric utility function for their analysis. They explored rationality of several economic variables including two measures of price level: CPI and PPI. The role of the forecast played consensus forecast of 30-40 participants of MMS[2]. Time span from the last quarter of 1977 till the end of 1993 was used.

Aggarwal et. al. interpreted rationality as forecast's unbiasness and additionally better predictive ability than simple autoregressive model. Rationality hypothesis was not rejected for both measures of the price level. The authors see their main contribution in the fact that they tested all series for stationarity and cointegration. The papers dealing with forecast rationality evaluation before neglected this step.

Granger (1999) derived theoretically-based properties which should be fulfilled by the rational forecast. These properties must be satisfied by the derivative of the loss function: unbiasness, absence of autocorrelation and absence of correlation with economic variables from the information set. Granger's approach is discussed in details in theoretical part.

Elliott, Timmermann, Komunjer (2005) considered a general shape of the loss function:

$$L(p, \alpha, e_{t+1}) = \left(\alpha + (1-2\alpha)1_{e_{t+1}<0}\right)\left|e_{t+1}\right|^{p}.$$

This specification includes quadratic and linear loss for the values p=2 and p=1 respectively. Moreover, $\alpha = \frac{1}{2}$ corresponds to symmetric loss function, whereas for other values of $\alpha \in (0; 1)$ the loss function is asymmetric.

The authors derive unbiased estimate of parameter α taking parameter p as given. The rationality test proposed in the article is equivalent to Hansen's J-test for overidentifying restrictions.

After that simulation results are provided. Firstly, Monte Carlo simulation was used to identify the size of the t-test $\hat{\alpha} = \alpha^{true}$. Different sample sizes and values of p were considered. Simulation results showed that the size of the test is acceptable. However, the results are slightly worse for quadratic loss functions than for linear. Not surprisingly that growth of the sample size improves the results.

For the second part of the simulation exercise the size of the rationality test was considered. The test performs better for symmetric loss function than for asymmetric.

[2] Money Market Services forecast data

Finally, proposed approach was implemented for IMF and OECD projections of the government budget deficit for G7 countries. The dataset contained forecasts for current year and the next year. OECD projections contained 24-27 observations whereas IMF projections contained 25 observations for each G7 country.

Econometric implementation of the proposed methodology helped to detect significant evidence of asymmetry of the loss functions for most of the countries for both datasets. After that composite test for rationality and symmetric loss function were made. In half of the cases null hypothesis is rejected. However, the results for asymmetric loss function are different. Almost all forecasts are consistent with rationality hypothesis under asymmetric loss function.

Both asymmetric and symmetric squared loss functions were considered in the article Capistran (2008) which is equivalent to p=2 in the loss function from Elliott, Timmermann, Komunjer (2005). The notion of rationality is treated as following properties of the derivative of the loss function: unbiasness, absence of serial correlation and absence of correlation with economic variables from the information set. Under symmetric squared loss these three conditions must be satisfied for the forecast error. However, under symmetric squared loss function the rationality hypothesis is rejected.

The author concluded that it is the consequence of the wrong functional form of the loss function. Consequently, asymmetric loss function was considered. OLS estimates of the asymmetry coefficient reject the hypothesis that it is equal to ½ which corresponds to symmetric loss. Moreover, GGM estimates of the model with asymmetric loss enables the authors to conclude that rationality hypothesis was not rejected.

Section 2. Theoretical part

In this section theoretical foundations of forecast rationality are reported. First subsection describes forecaster's program and its testable implications. Second subsections summarizes most commonly used in the literature types of loss functions.

2.1 Theoretical approach

Theoretical foundations used in this paper replicate the approach used in Granger (1999).

Consider the program of the forecaster. Let denote the forecast made by the forecaster at period t for the horizon h as $\pi^f_{t+h,t}$.

Let's assume that each forecaster has a convex loss function $L(e_{t+h,t})$, where $e_{t+h,t}$ is a forecast error defined as $e_{t+h,t} = \pi_{t+h} - \pi^f_{t+h,t}$ and π_{t+h} is a true value of inflation at period t+h.

Then the forecaster's projection is a solution of the following program:

$$\min_{\pi^f_{t+h}} E[L(\pi_{t+h} - \pi^f_{t+h})|I_n],$$

I_n is an information set available to the forecaster at the moment of building a projection.

When the underlying conditional distribution is known and let $P_{t,h} \equiv prob(\pi_{t+h} \leq x|I_n)$ then the program can be rewhritten as

$$\min_{\pi^f_{t+h}} \int L(\pi_{t+h} - \pi^f_{t+h}) dP_{t,h}$$

FOC:

$$\int L'(\pi_{t+h} - \pi^f_{t+h}) dP_{t,h} = 0$$

If $\tilde{\pi}^f_{t+h}$ is a solution of the problem then $\int L'(\pi_{t+h} - \tilde{\pi}^f_{t+h}) dP_{t,h} = 0$.

Let's introduce random variable $Z_{t+h} = L'\left(\pi_{t+h} - \tilde{\pi}^f_{t+h}\right) = L'(e_{t+h})$. Then three main properties of this variable are:

1. $E(Z_{t+h}|I_t) = 0;$
2. $E(f(X_{t-i})Z_{t+h}|I_t) = 0$, where $f(X_{t-i})$ is a function of any random variable with the realization from the information set;
3. $E(Z_{t+h}Z_{t+h-k}|I_t) = 0, where\ k > h.$

The first property follows from the first order condition derived above. The last two qualities are obtained from the properties of conditional expectation.

2.2 Types of loss function

Econometric literature devoted to the topic of rationality employs many types of loss functions. The most commonly used specification is a **quadratic loss function** (we will omit indexes for simplicity):

$$L(e) = e^2$$

Then the derivative of the loss function is $L'(e^2) = 2e$ and three properties could be reformulated as follows:

1. Unbiasness of the forecast error;
2. Absence of correlation between forecast error and any variable known at the moment of the forecast;
3. Absence of serial correlation in the forecast errors of the order higher than the forecasting horizon.

The intuitive interpretation of the results is a clear advantage of the model. However, the assumption that positive and negative forecast errors (which is equivalent to the forecasts lower and higher than the actual level of inflation respectively) are "punished" equivalently is not realistic. That is why it might be useful to consider asymmetric loss function.

One of the possible ways to model asymmetry is to use **asymmetric quadratic loss** function:

$$L\left(e_{t+h,t}, \varphi\right) = \begin{cases} \varphi e^2_{t+h,t}, e_{t+h,t} > 0 \\ (1-\varphi) e^2_{t+h,t}, e_{t+h,t} \leq 0 \end{cases} = \left(\varphi + (1-\varphi) 1_{e_{t+h,t} > 0}\right) \left|e_{t+h,t}\right|^2,$$

$$L'(e) = e - (1 - 2\varphi)|e|.$$

Parameter φ characterizes different "fine" for over- and under prediction. If a projection produced by the forecaster is greater than an actual value (positive forecast error) squared forecast error is multiplied by φ whereas a projection below an actual value (negative forecast error) enters the loss function multiplied by the coefficient $1 - \varphi,\ \ \varphi \in (0; 1)$. The case $\varphi = \frac{1}{2}$ is equivalent to the symmetric quadratic loss function discussed above. When $\varphi > \frac{1}{2}$ overprediction is more costly in terms of loss function than underprediction. The situation is opposite if $\varphi < \frac{1}{2}$.

In the paper Elliott, Timmermann, Komunjer (2005) a more general type of loss function was proposed:

$$L(p, \alpha, e_{t+1}) = \left(\alpha + (1-2\alpha)1_{e_{t+1}<0}\right)\left|e_{t+1}\right|^{p}. \quad (1)$$

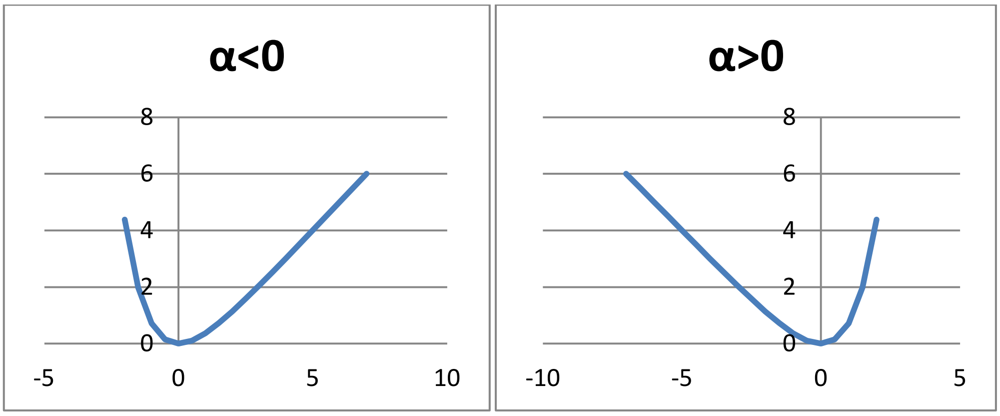
This type of loss function includes some of particular cases taken into consideration above. The value p=2 corresponds to quadratic loss, whereas p=1 to linear loss. Moreover, $\alpha = 0.5$ matches the case of symmetric loss and other parameter values to asymmetric loss.

Another approach to take possible asymmetry of the loss function into consideration is to use **Linex function**:

$$L\left(e_{t+h,t}\right) = e^{\alpha e_{t+h,t}} - \alpha e_{t+h,t} - 1,$$

$$L'\left(e_{t+h,t}\right) = \alpha e^{\alpha e_{t+h,t}} - \alpha.$$

If parameter $\alpha>0$ the function looks like exponent when the forecasting error is greater than zero and as line when the forecast error is negative (and vice versa when α<0).It is schematically demonstrated on the Picture 1.



Picture 1

Section 3. Data description

For this study we use Survey of Professional forecasters as source of data. It is important to emphasize that professional forecasters represent a very specific category of agents. They work in large banks, thinktanks, investment banks, consultancy and large corporations. This implies that forecasting economic variables is an important part of their job. Then we could anticipate that these agents bear reputational risks and, consequently, they are interested in producing an accurate forecast.

Moreover, professional forecasters have sufficient knowledge in economics. Presumably most of the firms participating in the survey have economic or econometric models to forecast at least some of economic variables. All of this means that forecasters might produce their projections on the basis of objective information. That is why we suppose that projections made by professional forecasters are perfect candidates for rationality tests.

To conduct empirical tests of the forecast rationality two datasets were used: Survey of Professional Forecasters conducted by Philadelphia FED and European Central Bank. The employment of datasets for different countries enables us to compare robustness of results.

3.1 Survey of professional forecasters US

For this study projections made by individual forecasters from the US survey of professional forecasters (SPF-US henceforth) were used. SPF-US is conducted by Federal Reserve Bank of Philadelphia form the second quarter of 1990 till present (it was conducted by ASA-NBER from the last quarter of 1968 till the first quarter of 1990). Individual forecasts are available in anonymous form.

The survey is held each quarter and each forecaster gives prediction for the previous quarter, current quarter and 1-4 quarters ahead. Moreover, predictions for the year when the survey is conducted and for two next years are done. The forecasters are asked to make a projection for a number of economic series including two measures of inflation: CPI and PCE.

For the purpose of this study CPI forecasts were used. Only rolling horizon was considered to increase the number of available observations because for current quarter and 1-4 quarters ahead each calendar year contains 4 actual observations for each horizon and only one for yearly forecasts. Moreover, the forecasts for the previous quarter were excluded because of very low percent of answers to this question among forecasters. To sum up, forecasts 0-4 quarters ahead were used in this work (i. e. forecast horizon h=0, 1,…,4).

According to the SPF documentation available on FRB's website, the participants of the survey are asked to forecast seasonally adjusted annual rate. Quarterly forecasts are annualized quarter-over-quarter percent changes. This definition is a bit vague because forecasters might use different routines for the seasonal adjustment of the time series (Tramo/Seats, X-12 and so on). That is why it is not clear CPI data with what type of seasonal adjustment should be used as actual. To avoid this problem the actual data for CPI was taken from "Error Statistics for the Survey of Professional Forecasters" which is also available on the website of FRB of Philadelphia.

Taking into account possible problems with forecasters' ids during the time when the survey was conducted by ASA-NBER only forecasts for the period from 1981 were taken into consideration. In order to have enough observations to estimate models only forecasters with at least 30 projections were included into the sample. Finally, rationality test was done for 45 forecasters.

In addition to CPI data the information on key US economic indicators was used from OECD database. Following exogenous variables were used: industrial production, retail trade, unemployment rate, broad money, overnight interbank rate, 3 month interbank rate, long-term interest rate, export, import, current account in percent to GDP, GDP. Industrial production, retail trade, broad money, export, import and GDP are measured as seasonally adjusted growth to the previous period. Unemployment rate and current account in percent to GDP are seasonally adjusted.

3.2 Survey of professional forecasters ECB

The second dataset used for this study consists of projections made by individual forecasters from the European Central Bank survey of professional forecasters (SPF-ECB henceforth). This survey is conducted form the first quarter 1999 till the present time on the quarterly bases. The questionnaires contain questions about inflation, GDP and unemployment as well as participant's assumptions about ECB's interest rate, oil prices, USD/EURO exchange rate and labour costs.

In the survey inflation is seen as euro area inflation measured by Harmonized Index of Consumer Prices (HICP) published by Eurostat. Each quarter participants are asked to make inflation projections for current calendar year, two following calendar years, long term forecast (four calendar years in the first and second quarters and five calendar years in the third and fourth quarter) and two rolling horizon forecasts one and two years ahead from the last available monthly observation. However, the design of the questionnaire was different until the second quarter of 2001: a question about long term forecast was present only in the questionnaires for the first quarter. Moreover, rolling horizon forecast five years ahead was present.

SPF-ECB has smaller number of observations comparing to SPF-US. Forecasts for the calendar year do not provide enough observations for econometric analysis: only 15 actual points for the years 1999-2013. That is why only rolling horizon forecasts were used for this study. 5 years ahead rolling horizon forecasts were asked only in 9 survey rounds and a very small percent of forecasters provided their projection for this horizon. For the reasons mentioned above only one and two years ahead rolling horizon forecasts were used for econometric rationality tests.

Rolling horizon forecasts represent the forecast of inflation for the month one and two years ahead from the last available observation. The structure of the forecast is represented in Table 1.

Table 1

Date of survey	**Last available observation**	**1 year ahead forecast**	**2 years ahead forecast**
1Q1999	Dec1998	Dec1999	Dec2000
2Q1999	Mar1999	Mar2000	Mar2001
3Q1999	Jun1999	Jun2000	Jun2001
4Q1999	Sep1999	Sep2000	Sep2001
1Q2000	Dec1999	Dec2000	Dec2001
2Q2000	Mar2000	Mar2001	Mar2001
…	…	…	…

Using rolling horizon forecasts enables us to obtain two time series of forecasts (for horizon h=1, 2 years ahead) for each forecaster. Only 38 forecasters who have at least 10 non-missing observations for each of forecast horizons were taken into consideration. Of cause, 10 observations might not be enough for running econometric procedures. However, the balance between number of forecasters under consideration and number of observations for each of them was important for us. Only 7 forecasters have at least 14 nonempty observations for each of the forecasting horizons. Moreover, none of survey participants has 15 nonempty observations for both forecast horizons.

The actual data on HICP chain index was taken from Eurostat. Monthly data was used and inflation was computed relative to the same month of the previous year and to the same month of the year two years ago for forecast horizon one and two years ahead respectively. After that the observations corresponding to March, June, September and December were used as actual data.

Finally, the same set of exogenous variables was used as in the case with SPF-US forecasts. Key economic indicators for Euro Area were used from OECD database. The indicators were measured as seasonally adjusted growth to the same period of the previous year and growth to the same period 2 years ago for forecast horizon equal 1 and 2 years respectively. This was done in order to make exogenous variables comparable with forecasts.

Section 4. Empirical part

Empirical work was conducted in two stages. On the first stage individual forecasts were tested for rationality employing different techniques and types of rationality functions. After that each of individual forecasts was marked as “rational” or “not rational” according to each way of rationality evaluation. Then average forecasts were computed for rational and non-rational subgroups as well as for all forecasters. After that Diebold-Mariano test was used for pairwise comparison of predictive ability between these three groups. All procedures described above were made both for SPF-US and SPF-ECB forecasters.

For SPF-US experts time span from first quarter of 1981 till fourth quarter 2013 was used to conduct a test for rationality of individual forecasts as well as to compare quality of forecasts between the groups. The attempt to evaluate two stages on different subsamples was made. However, it leads to the decrease in the number of forecasters, because about one fourth of the individual forecasters started participating in the survey in 2006 or later. Nevertheless, the results for rationality test for remaining part of the sample are robust. Finally, using only subsample for comparison of forecasts’ accuracy lead to insufficient number of observations to compute Diebold-Mariano statistic. That is why it was decided to estimate both stages using all available observations.

Rationality tests for individual forecasters and comparison of the forecast accuracy for SPF-ECB experts was done using the period from the first quarter of 2000 (2001 for forecasts two years ahead) till fourth quarter 2013.

4.1 Rationality test for individual forecasts

After primary work our dataset could be summarized as follows. We have 54 SPF-US individual forecasts for forecast horizon 0-4 quarters ahead. These projections cover time span from third quarter 1981 till fourth quarter 2013 with on average 51 non-missing observations.

SPF-ECB database has fewer observations than SPF-US. For empirical tests we used projections made by 38 SPF-ECB forecasters for forecast horizon 1 and 2 years ahead. Forecasts for the period December 1999 – December 2013 are available for us. On average SPF-ECB forecasters have 31 non-missing observations.

Describes above two datasets were tested for rationality. In this work three types of loss functions were used: symmetric squared, asymmetric squared and linex. In the following subsections econometric tests for rationality for each of these functions are described.

4.1.1 OLS estimation

From the theoretical point of view all exogenous variables should be included into regression simultaneously. However, for the sake of increasing number of degrees of freedom only one exogenous variable was included into regression. Moreover, for the same reason only one lag of forecast error and exogenous variable was included into all of the models. In other words, each of the models was estimated for each exogenous variable.

In other words, our approach consists in testing the properties of forecast errors using only subset of available information. We understand that it is not equivalent to the test which employs all available information but our datasets do not contain sufficient observations to employ more theoretically solid approach.

For the symmetric loss function the following regression is estimated:

$$e_{t+h,t} = \alpha + \beta e_{t-1} + \gamma x_{t-1} + \varepsilon_t, \quad (3)$$

Then the hypothesis is tested: $H_0 = \alpha = \beta \quad = \gamma \quad = 0$.

The forecaster i is called "rational" if H_0 is not rejected on the 10% significance level at least for 90% of exogenous variables. In the tables all results corresponding to the squared symmetric loss function are labeled **"symsq"**.

None of the experts from the SPF-ECB database was classified as "rational" according to symmetric squared loss function. That is why this type of function was not used in the next stage for SPF-ECB data.

In case of asymmetric quadratic loss function two-step procedure was used. On the first stage the asymmetry parameter is estimated from the model:

$$e_{t+h,t} = \alpha + (1 - 2\varphi)\left|e_{t+h,t}\right| + \gamma \quad X_{i,t-1} + \varepsilon_t.$$

After that the derivative of the loss function is computed. At the second stage the regression similar to (3) is estimated, in which forecast error is replaced with the derivative of the loss function. Finally, the same hypothesis is tested.

In this case the forecaster was called "rational" if two conditions were satisfied simultaneously. Firstly, hypothesis that asymmetry parameter is equal to ½ should be rejected on 10% significance level which is equivalent to admit that the forecaster has asymmetric squared utility function. Secondly, hypothesis about simultaneous equality to zero of all the coefficients in rationality test regression should not be rejected on 10% significance level.

Unfortunately, the application of this model to the data showed the model cannot adequately describe the behavior of forecasters both from SPF-US and SPF-ECB database. None of the forecasters could be classified as "rational" according to this rule. For this reason this model was not used on the second stage.

The situation is a bit more complicated with Linex loss function. Given the value of parameter the procedure similar to the one described for the quadratic loss function can be applied. However, there is no proposed in the literature method for estimation of the parameter. In addition to that, ML estimation is complicated because of the small number of observations and without certainty in concavity of the likelihood. That is why the grid search was used.

For each forecaster and each exogenous variable the rationality test was made for each value of the parameter of the linex function (alpha) between -3 and 3 (except for 0) with step 0.1. After that rationality was defined in three different ways. Firstly, p-value for the testing of the rationality hypothesis was averaged for each alpha. After that alpha which corresponds to the maximal average p-value was chosen. Then model was re-estimated with the use of calculated alpha. The forecaster was classified as "rational" if for calculated alpha H_0 is not rejected on the 10% significance level at least for 70% of exogenous variables. This type of model is marked **"linex_avmax"** in the tables.

Secondly, for each alpha the minimal p-value was extracted. Then alpha was chosen corresponding to the largest of these p-values. Again, model was re-estimated with obtained alpha and the same notion for rationality was used as in the previous case. This definition of rationality is labeled **"linex_maxmin"** in results.

Finally, the forecaster was admitted rational if for her existed at least one value of alpha such that for all exogenous variables p-value was greater than significance level. If there were more than one such value of alpha they were averaged. This notion of rationality is labeled as **"linex_bin"** in the tables.

To make our methodology more clear we illustrate it with Table 2. Table 2 depicts rationality test's results for one of the SPF-US forecasters. Columns represent exogenous variables: production, retail trade, unemployment rate, broad money, overnight interbank rate, 3 month interbank rate, long-term interest rate, export, import, current account in percent to GDP, GDP.

For each type of the loss functions p-values testing null hypothesis about forecast rationality are reported. All p-values larger than 10% are colored with grey.

For **symmetric squared** loss function null hypothesis is not rejected for 4 variables. Consequently, the forecaster is classified as non-rational according to this type of loss function.

Table contains additional information for **asymmetric squared** loss function. Along with p-value for rationality test, estimate of the asymmetry parameter and hypothesis about equality of this parameter to ½ (symmetric loss function) are reported. For the forecaster hypothesis about symmetric squared loss function is not rejected for all exogenous variables. As a consequence, she is classified as "non-rational".

Speaking about linex function, results of the rationality test for each parameter alpha from the greed are reported. Moreover, three types of calculating parameter alpha common for all exogenous variables are demonstrated.

For **linex_avmax** average p-value for each parameter value is calculated then maximum is defined. For this forecaster it corresponds to $\alpha=3$. Then models are re-estimated with this value of parameter. For re-estimated models Ho about forecast rationality is not rejected for all variables and forecaster is marked as "rational".

In the case of **linex_maxmin** minimal p-value for each parameter value is calculated. After that maximum of these values is computed (again, it corresponds to $\alpha=3$). As in the previous case, according to re-estimated models forecaster is classified as "rational".

Finally we consider **linex_bin** type. We are interested in parameter values for which null hypothesis about forecaster's rationality is not rejected for all exogenous variables. For this forecasters it corresponds to the values $\alpha=1.8,\ldots,3$. As for the forecaster exists at least one such parameter value she is classified as "rational". Finally, α_{bin} is computed as average and is equal to 2.4.

Table 2

	prod	sales	unemp	m2	i_o	i_s	i_l	ex	im	ca_to_gdp	gdp	average	minimum
	Symmetric quadratic loss												
p-value	0.099	0.126	0.126	0.042	0.014	0.023	0.046	0.086	0.065	0.104	0.111		
	Aymmetric quadratic loss												
p-value	0.439	0.616	0.601	0.132	0.048	0.088	0.193	0.502	0.241	0.513	0.448		
phi	0.615	0.604	0.609	0.634	0.631	0.627	0.623	0.585	0.626	0.606	0.627		
H0: phi=1/2	0.409	0.418	0.414	0.394	0.397	0.400	0.403	0.432	0.401	0.417	0.400		
	LINEX											average	minimum
-3.0	0.059	0.556	0.266	0.587	0.057	0.065	0.067	0.443	0.159	0.219	0.001	0.225	0.001
-2.9	0.059	0.556	0.265	0.586	0.056	0.065	0.067	0.442	0.159	0.219	0.001	0.225	0.001
-2.8	0.059	0.555	0.265	0.584	0.056	0.065	0.067	0.442	0.158	0.218	0.001	0.224	0.001
-2.7	0.059	0.554	0.264	0.583	0.056	0.064	0.066	0.441	0.158	0.217	0.001	0.224	0.001
-2.6	0.059	0.552	0.263	0.581	0.056	0.064	0.066	0.440	0.157	0.216	0.001	0.223	0.001
-2.5	0.058	0.551	0.261	0.578	0.055	0.064	0.065	0.439	0.156	0.215	0.001	0.222	0.001
-2.4	0.058	0.548	0.260	0.574	0.054	0.063	0.065	0.438	0.155	0.214	0.001	0.221	0.001
-2.3	0.058	0.546	0.257	0.570	0.054	0.062	0.064	0.436	0.154	0.212	0.001	0.219	0.001
-2.2	0.057	0.542	0.255	0.564	0.053	0.061	0.063	0.433	0.152	0.210	0.001	0.217	0.001
-2.1	0.057	0.537	0.251	0.557	0.052	0.060	0.062	0.430	0.150	0.208	0.001	0.215	0.001
-2.0	0.056	0.531	0.247	0.548	0.050	0.058	0.060	0.427	0.148	0.205	0.001	0.212	0.001
-1.9	0.055	0.524	0.242	0.536	0.048	0.056	0.059	0.422	0.145	0.201	0.001	0.208	0.001
-1.8	0.054	0.515	0.236	0.522	0.046	0.054	0.057	0.416	0.141	0.196	0.001	0.203	0.001
-1.7	0.052	0.504	0.229	0.504	0.044	0.051	0.054	0.409	0.136	0.191	0.001	0.198	0.001
-1.6	0.051	0.491	0.220	0.483	0.041	0.048	0.051	0.401	0.130	0.185	0.001	0.191	0.001
-1.5	0.049	0.476	0.211	0.456	0.037	0.044	0.048	0.391	0.123	0.178	0.001	0.183	0.001
-1.4	0.047	0.459	0.199	0.425	0.033	0.040	0.044	0.380	0.115	0.171	0.001	0.174	0.001
-1.3	0.045	0.440	0.188	0.388	0.029	0.036	0.040	0.368	0.107	0.163	0.001	0.164	0.001
-1.2	0.043	0.420	0.175	0.348	0.025	0.031	0.036	0.356	0.098	0.156	0.001	0.154	0.001
-1.1	0.042	0.401	0.164	0.304	0.021	0.027	0.032	0.347	0.089	0.151	0.001	0.143	0.001
-1.0	0.041	0.386	0.155	0.260	0.017	0.023	0.029	0.343	0.080	0.148	0.001	0.135	0.001
-0.9	0.043	0.381	0.151	0.219	0.014	0.020	0.026	0.349	0.074	0.151	0.001	0.130	0.001
-0.8	0.048	0.391	0.157	0.184	0.012	0.018	0.026	0.375	0.071	0.164	0.002	0.132	0.002
-0.7	0.061	0.425	0.178	0.159	0.011	0.018	0.028	0.435	0.072	0.195	0.004	0.144	0.004
-0.6	0.087	0.495	0.229	0.146	0.011	0.020	0.035	0.544	0.082	0.256	0.009	0.174	0.009
-0.5	0.141	0.603	0.326	0.144	0.014	0.027	0.052	0.704	0.102	0.359	0.024	0.227	0.014
-0.4	0.230	0.713	0.471	0.150	0.020	0.040	0.084	0.855	0.136	0.491	0.071	0.296	0.020
-0.3	0.321	0.740	0.584	0.150	0.029	0.056	0.123	0.879	0.171	0.568	0.169	0.345	0.029
-0.2	0.326	0.598	0.536	0.127	0.033	0.061	0.135	0.674	0.174	0.484	0.256	0.309	0.033
-0.1	0.223	0.335	0.381	0.084	0.026	0.047	0.120	0.314	0.154	0.292	0.225	0.200	0.026
0.1	0.037	0.040	0.080	0.020	0.009	0.015	0.045	0.019	0.034	0.039	0.041	0.034	0.009
0.2	0.013	0.013	0.014	0.009	0.003	0.004	0.008	0.004	0.012	0.011	0.013	0.009	0.003
0.3	0.006	0.005	0.006	0.005	0.001	0.002	0.004	0.001	0.006	0.005	0.005	0.004	0.001
0.4	0.004	0.003	0.004	0.004	0.001	0.001	0.002	0.001	0.004	0.003	0.003	0.003	0.001
0.5	0.004	0.003	0.004	0.004	0.001	0.002	0.002	0.001	0.004	0.003	0.003	0.003	0.001
0.6	0.006	0.004	0.005	0.006	0.002	0.002	0.003	0.001	0.006	0.003	0.004	0.004	0.001
0.7	0.010	0.008	0.009	0.010	0.003	0.004	0.006	0.002	0.010	0.004	0.007	0.007	0.002
0.8	0.019	0.014	0.019	0.020	0.007	0.008	0.011	0.004	0.020	0.007	0.014	0.013	0.004
0.9	0.038	0.028	0.037	0.038	0.013	0.015	0.021	0.011	0.039	0.012	0.028	0.025	0.011
1.0	0.070	0.052	0.069	0.069	0.024	0.026	0.037	0.025	0.071	0.019	0.053	0.047	0.019
1.1	0.117	0.089	0.117	0.115	0.041	0.044	0.059	0.051	0.118	0.028	0.092	0.079	0.028
1.2	0.179	0.139	0.179	0.175	0.063	0.065	0.088	0.091	0.179	0.039	0.145	0.122	0.039
1.3	0.249	0.198	0.250	0.243	0.088	0.090	0.120	0.143	0.249	0.051	0.209	0.172	0.051
1.4	0.323	0.261	0.323	0.315	0.114	0.115	0.152	0.204	0.322	0.064	0.277	0.224	0.064
1.5	0.394	0.324	0.393	0.384	0.139	0.138	0.183	0.268	0.391	0.076	0.344	0.276	0.076
1.6	0.458	0.383	0.456	0.447	0.162	0.159	0.210	0.331	0.455	0.086	0.407	0.323	0.086
1.7	0.514	0.436	0.511	0.502	0.182	0.177	0.233	0.389	0.510	0.096	0.462	0.365	0.096
1.8	0.561	0.483	0.558	0.550	0.199	0.192	0.253	0.441	0.557	0.104	0.510	0.401	0.104
1.9	0.601	0.523	0.596	0.590	0.214	0.204	0.269	0.486	0.596	0.111	0.550	0.431	0.111
2.0	0.634	0.556	0.628	0.622	0.225	0.214	0.282	0.525	0.628	0.117	0.584	0.456	0.117
2.1	0.660	0.585	0.654	0.650	0.235	0.222	0.293	0.557	0.654	0.123	0.612	0.477	0.123
2.2	0.682	0.609	0.677	0.672	0.243	0.228	0.301	0.584	0.675	0.127	0.635	0.494	0.127
2.3	0.700	0.629	0.694	0.690	0.249	0.233	0.308	0.607	0.693	0.131	0.654	0.508	0.131
2.4	0.715	0.645	0.708	0.705	0.254	0.237	0.314	0.626	0.707	0.134	0.670	0.519	0.134
2.5	0.727	0.658	0.720	0.718	0.258	0.240	0.318	0.641	0.718	0.136	0.683	0.529	0.136
2.6	0.737	0.670	0.729	0.728	0.261	0.242	0.322	0.654	0.728	0.139	0.694	0.537	0.139
2.7	0.745	0.679	0.737	0.736	0.264	0.244	0.325	0.665	0.736	0.141	0.702	0.543	0.141
2.8	0.752	0.687	0.743	0.743	0.266	0.246	0.327	0.675	0.742	0.142	0.710	0.548	0.142
2.9	0.757	0.694	0.749	0.749	0.268	0.247	0.329	0.682	0.747	0.144	0.716	0.553	0.144
3.0	0.762	0.699	0.753	0.754	0.269	0.248	0.331	0.689	0.752	0.145	0.721	0.557	0.145
α_{avmax}	3	3	3	3	3	3	3	3	3	3	3	α_{avmax}=3	
p-value	0.762	0.699	0.753	0.754	0.269	0.248	0.331	0.689	0.752	0.145	0.721		
α_{maxmin}	3	3	3	3	3	3	3	3	3	3	3	α_{maxmin}=3	
p-value	0.762	0.699	0.753	0.754	0.269	0.248	0.331	0.689	0.752	0.145	0.721		
α_{bin}	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	α_{bin}=2.4	
p-value	0.715	0.645	0.708	0.705	0.254	0.237	0.314	0.626	0.707	0.134	0.670	Re-estimated models for α_{bin}=3	

Ho is not rejected for 4 exogenous variables → forecaster is not rational according to **SYMSQ**

Ho about the symmetry of loss function is not rejected for all variables → forecaster is not rational for **ASYMSQ**

Average p-value for each α

Minimal p-value for each α

Values of α for which Ho is not rejected for all exogenous variables → the forecaster is rational according to **LINEX_BIN**.

Maximal average p-value corresponds to α_{avmax}=3

Largest minimal p-value corresponds to α_{maxmin}=3

Re-estimated models for α_{avmax}=3. Ho is not rejected for all exogenous variables → forecaster is rational **LINEX_AVMAX**

α_{maxmin}=3. Ho is not rejected for all exogenous variables → forecaster is rational **LINEX_MAXMIN**

4.1.2 GMM estimation

In spite of the fact that earlier papers used OLS estimation rationality test could be conducted in a natural way with the use of Generalized Method of Moments (GMM). The approach is similar to one usually used for estimating macroeconomic models with rational expectations.

The first order condition of the forecaster's program is $E(L'(e_{t+h})|I_t) = 0$. Then using instruments W_t and law of iterated expectations: $E([W_t L'(e_{t+h})] = E[E(W_t L'(e_{t+h})|I_t)] = E[W_t E(L'(e_{t+h})|I_t)] = 0$.

In other words, moment conditions are $E([W_t L'(e_{t+h})] = 0$.

In the paper Elliott, Timmermann, Komunjer (2005) was shown that under a set of constraints and taken the value of p given the consistent estimate of parameter alpha for the loss function (1) could be found as

$$\widehat{\alpha_T} = \frac{\left[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} w_t |e_{t+1}|^{p-1}\right]' \widehat{S^{-1}} \left[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} w_t 1_{e_{t+1}<0} |e_{t+1}|^{p-1}\right]}{\left[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} w_t |e_{t+1}|^{p-1}\right]' \widehat{S^{-1}} \left[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} w_t |e_{t+1}|^{p-1}\right]}, \qquad (2)$$

where w_t – the vector of instruments, T – number of observed forecast errors. $\hat{S}$ could be found as $\hat{S}(\widehat{\alpha_T}) = T^{-1}\sum_{t=\tau}^{T+\tau-1} w_t w'_t 1_{e_{t+1}<0} |e_{t+1}|^{2p-2}$.

In the paper Elliott, Timmermann, Komunjer (2005) no reference to GMM estimation is made. However, Appendix A shows the equivalence of these two approaches.

As estimates of α_T and $S(\widehat{\alpha_T})$ are interdependent the trick usual for GMM estimation could be applied. On the first stage S is chosen as identity matrix and estimate of alpha is obtained. Then $\hat{S}(\widehat{\alpha_0})$ can be calculated. After that these steps are iterated until α is equal to the estimate obtained on the previous step.

Rationality test proposed in the paper is equivalent to the Hansen's J-test for over identifying restrictions:

$$J = \frac{1}{T}\left[\left(\sum_{t=\tau}^{T+\tau-1} w_t (1_{e_{t+1}<0} - \widehat{\alpha_T}) |e_{t+1}|^{p-1}\right)' \widehat{S^{-1}} \left(\sum_{t=\tau}^{T+\tau-1} w_t (1_{e_{t+1}<0} - \widehat{\alpha_T}) |e_{t+1}|^{p-1}\right)\right] \sim \chi^2_{d-1},$$

where d is a number of instruments.

In this research a number of models with this type of loss function were estimated. For linear and squared loss functions parameter α was estimated with formula (2). After that rationality test was conducted and all forecasters were labeled as "rational" or "non-rational". In addition to that, for both of the cases α was set equal to ½ (which corresponds to symmetric loss) and rationality test was conducted for given parameter value. Again, the classification of forecasters was done.

The role of instruments played constant, last known at the moment of producing the forecast forecasting error and lags of all exogenous variables. For SPF-ECB data no forecasting error was included because of irregular observations.

Elliott, Timmermann, Komunjer (2005) was proved that it is not necessary to use all information used by the forecaster as instruments. Instead, only a subset of the information is enough to recover the parameter of the loss function. It is a very strong theoretical result because not knowing private information probably used by the forecaster we can limit our analysis only to publically available variables.

4.2 Robustness of forecasters' rationality to the forecast horizon

A more theoretically solid approach would be to conduct each of the above-described rationality tests for particular forecaster for all forecast horizons simultaneously. In other words, to use

$e_t^i = \left(e_0^i, e_1^i, e_2^i, e_3^i, e_4^i\right)'$ for each forecaster i to run rationality tests for all forecasting horizons simultaneously. However, large number of missing observations in forecast time series resulted in insufficient number of complete cases to apply this method.

That is why it is necessary to analyze how robust are results of the rationality test for each of the forecasters with respect to the forecast horizon. The results of this analysis are given in Table 3 for SPF-US data and in Table 4 for SPF-ECB.

In Table 1 for each of 54 SPF-US forecaster's stability of their division into "rational" and "non-rational" subsamples is analyzed. In the table the forecaster is marked "**NR**" if for a given method of estimation and given type of the loss function she was classified as "non-rational" for all forecast horizons (except for the cases where the model did not converge). Similarly, the forecaster is marked "**R**" if for a given method of estimation and given type of the loss function she was classified as "rational" for all forecast horizons (except for the cases where the model did not converge). If for a particular forecaster none of the models of a given type converged she is labeled "**NaN**".

Table 3

Forecaster ID	OLS estimation				GMM estimation			
	symsq	linex_avmax	linex_maxmin	linex_bin	symsq	asymsq	symlin	asymlin
1	NR				NR	NR		NR
2					NR	NR	NR	NR
3								NR
4					NR	NR	NR	NR
5	NR							NR
6					NR	NR		NR
7					NR	NR		NR
8	NR				NR		NR	
9		R	R	R	NR		NR	NR
10					NR			NR
11								NR
12	NR				NR	NR	NR	NR
13					NR	NR	NR	NR
14			R	R	NR		NR	NR
15		R	R	R				NR
16	NR				NR		NR	NR
17		R	R	R	NR			NR
18			R	R	NR	NR	NR	NR
19		R	R	R	NR	NR	NR	NR
20	NR				NR	NR	NR	NR
21			R	R	NR		NR	NR
22		R	R	R				NR
23		R	R	R	NR		NR	NR
24			R	R	NR	NR	NR	NR
25							NR	NR
26		R	R	R	NR			NR
27	NR				NR	NR		NR
28			R	R				NR
29	NR		R	R	NR	NR		NR
30	NR		R	R			NR	NR
31		R	R	R				NR
32	R	R	R	R		NR		NR
33	R		R	R				NR
34			R	R				NR
35	NR		R	R	NR	NR	NR	NaN
36		R	R	R	NR	NR		NR
37	NR		R	R	NR	NR	NR	
38	NR	R	R	R	NR		NR	NR
39	R	R	R	R				NR
40		R	R	R	NR	NR	NR	NR
41		R	R	R				NR
42		R	R	R	R			NR
43			R	R				
44		R	R	R		NR		NR
45			R	R				NR
46			R	R				NR
47					NR	NR	NR	NaN
48			R	R				NR
49	R	R	R	R				NR
50	R	R	R	R				
51	R	R	R	R	NR			NR
52	R	R	R	R				
53		R	R	R		NR		NaN
54	R	R	R	R				

It could be seen from the Table 3 that OLS estimation is more prawn to classifying forecasters as "rational" whereas the situation with GMM estimates is opposite. For a particular forecaster the stability classification is heavily dependent on the type of loss function. For example, forecaster 38 is classified as "non-rational" under the assumption of symmetric squared loss function with both OLS and GMM. Other types of loss functions employing OLS estimation classify this forecaster as "rational", whereas for GMM estimation she is classified either as "non-rational" or for asymmetric squared loss function classification is unstable.

Table 4 represents stability of classification for SPF-ECB forecasters. Notations are the same as in Table 3: forecaster is marked "**NR**" if for a given method of estimation and given type of the loss function she was classified as "non-rational" for all forecast horizons (except for the cases where the model did not converge). Similarly, the forecaster is marked "**R**" if for a given method of estimation and given type of the loss function she was classified as "rational" for all forecast horizons (except for the cases where the model did not converge). If for a particular forecaster none of the models of a given type converged she is labeled "**NaN**".

Table 4

	OLS estimation			*GMM estimation*			
Forecaster ID	**linex_avmax**	**linex_maxmin**	**linex_bin**	**asymlin**	**symsq**	**asymsq**	**symlin**
1				NR		NR	
2	NR	NR	NR	NR	R	NR	R
3	NR	NR	NR	NR		NR	
4	NR	NR	NR	NR	NR	NR	NR
5				NR	NR	NR	NR
6			NR	NR	NR	NR	NR
7	NR	NR	NR	NR	R	NR	R
8				NR		NR	R
9				NR	NR	NR	NR
10				NR	NaN	NR	NaN
11	NR	NR	NR	NR	NR	NR	NR
12	NR	NR	NR	NR	NR	NR	NR
13			NR	NR		NR	
14	R	R	NR	NR	NR	NR	NR
15	NR	NR	NR		NaN		NaN
16	NR	NR	NR	NR		NR	
17	NR	NR	NR	NR	NR	NR	NR
18	R	R	R	NR		NR	NR
19				NR		NR	
20				NR		NR	
21	NR	NR	NR	NR		NR	
22	NR		NR	NR		NR	
23	NR	NR	NR	NR	NR	NR	NR
24	NR	NR	NR	NR		NR	R
25	NR	NR	NR	NR	R	NR	R
26				NR	NR	NR	NR
27	NR	NR	NR	NR		NR	
28	NR	NR	NR	NR	NR	NR	NR
29	NR	NR	NR	NR	NR	NR	NR
30	NR	NR	NR	NR	NR	NR	NR
31	NR	NR	NR	NR	NaN	NR	NaN
32	NR	NR	NR	NR		NR	
33	NR	NR	NR	NR	NR	NR	NR
34				R	NaN	NR	NaN
35	NR	NR	NR	NR	R	NR	NR
36				NR		NR	
37	NR	NR	NR	NR		NR	
38	NR	NR	NR		R		NR

Unlike the situation with SPF-US forecasts OLS estimation for SPF-ECB experts classifies forecaster as "non-rational" in the majority of cases. This fact could not be explained by small number of observations: small power of the test results in the inability to reject null hypothesis. However, the situation is opposite heir. These results suggest that there exist some differences between US and ECB experts. For instance, use of different models. This might be an interesting research direction. However, to conduct this type of research more information is needed which is not available for public.

Speaking about GMM estimation for SPF-ECB data it could be seen from Table 4 that a number of observations resulted in non-convergence of the model in many cases (it is marked as "**NaN**").

Moreover, it is quite surprising that using GMM estimation and symmetric squared loss function all of the forecasters (for whom the algorithm converged) were classified as "rational" whereas for OLS estimation with the same type of loss function all of them mere marked as "non-rational".

4.3 Comparing predictive ability

After classification of the individual forecasters into two groups forecaster's accuracy was compared. Two groups were averaged in two ways: mean and median were used. In addition to that, forecasts obtained from two groups were compared with simple mean and median calculated using all available forecasters. After that forecast errors were computed as the difference between actual and forecast values.

There are a variety of tests for the comparison of the predictive ability of the models. The description of the majority of methods could be found in West (2006). For the purpose of this study we are interested in model-free tests as models used by forecasters are not known. This approach is not widely-used in economics because the researcher is usually interested in the evaluation of the predictive behavior of the estimated model.

Taking into account small sample size non-parametric tests were not used for this study as they are based on the asymptotic inference. Moreover, the variety of types of the loss function under consideration does not allow us to use tests relying on the assumption of the quadratic loss function, such that Morgan-Granger-Newbold test and Meese-Rogoff test.

Diebold-Mariano test (Diebold and Mariano 1995) proposed test statistic which does not rely on the assumption of quadratic loss function and therefore could be used for forecast comparison under general type of loss function. Moreover, following assumptions about forecast errors were relaxed: zero-mean, Gaussian, serially-uncorrelated and contemporaneously uncorrelated. However, simulations in the paper made evident the fact that the proposed test is seriously over-sized for small samples and in the case of multi-step ahead predictions.

These drawbacks of Diebold-Mariano test were partially corrected in Harvey, Leybourne and Newbold (1997). They derived small-sample adjustment for DM statistics. Simulation in the paper illustrated that modified DM statistics improved the size of the test for moderate number of observations. However, it is necessary to point out that in both papers simulations were conducted for the case of the symmetric loss function. Nevertheless, squared loss function assumption was not used for the derivation of neither DM nor modified DM statistics.

Summing up, to compare relative forecast accuracy modified Diebold-Mariano test proposed in Harvey, Leybourne and Newbold (1997) was used. Taking into account not very large sample size finite sample version of the test was used. Test statistics is as follows:

$$DM = \frac{\bar{d}}{\sqrt{((\gamma_0 + 2\sum \gamma_i)(H + 1 - 2j + H^{-1}(j-1)j)H^{-1}}},$$

where $\bar{d}$ is a "distance" between two forecast errors calculated on the basis of the chosen type of loss function calculated as $\bar{d} = \sum_{i=1}^{H}(L(e_i^1) - L(e_i^2))$,

γ_i is autocovariance of *d* at displacement *i*,

j – forecast horizon,

H – number of observations.

The hull hypothesis is that two models have equal predictive ability. The alternative hypothesis is that the first model is more accurate than the second one. Alternative hypothesis for three cases under consideration are:

- Rational VS Non-Rational:
  H_a: Forecasts of rational are more accurate;
- Rational VS All:
  H_a: Forecasts of rational are more accurate;
- Non-Rational VS All:
  H_a: Forecasts of non-rational are more accurate.

DM statistics has Student t-distribution with H-1 degrees of freedom (one-sided p-value was used).

4.4 Rationality of consensus forecast

In this subsection we concentrated on consensus forecasts instead of individual projections. The reasons for conducting rationality test for aggregate forecast are twofold. Firstly, consensus forecasts are more important for market participants who use them to form expectations about future variables. Some organizations that conduct surveys of professional forecasters do not provide individual forecasts and make available for the public only average projections.

Secondly, rationality test for aggregate forecast addresses a very important issue in economics: possibility to aggregate agents. From previous sections we saw that some forecasters are rational according to some types of the loss function. So, it is not clear whether the notion of rationality (in terms of loss function) is applicable on the individual level. As a consequence, we want to test the validity of rationality hypothesis on the aggregate level. Here we try to understand whether all these agents could be represented by "average forecaster" with her own loss function who acts rationally.

Table 5 summarizes results of rationality test for SPF-US consensus forecasts and Table 6 – for SPF-ECB. Aggregate forecast is calculated in two ways: mean and median of individual projections. Forecast is marked "**NR**" if for a given method of estimation and given type of the loss function it was classified as "non-rational" for all forecast horizons. Similarly, the forecast is marked "**R**" if for a given method of estimation and given type of the loss function it was classified as "rational" for all forecast horizons.

Table 5. Rationality test for SPF-US consensus forecast

Consensus forecast	OLS estimation				GMM estimation			
	symsq	linex_avmax	linex_maxmin	linex_bin	symsq	asymsq	symlin	asymlin
mean		R	R	R				
median		R	R	R			NR	NR

Table 6. Rationality test for SPF-ECB consensus forecast

Consensus forecast	OLS estimation			GMM estimation			
	linex_avmax	linex_maxmin	linex_bin	symsq	asymsq	symlin	asymlin
mean	NR	NR	NR	NR		NR	
median	NR	NR	NR	NR		NR	

SPF-US consensus forecasts are rational for linex loss function. However, more theoretically consistent GMM approach classifies both SPF-US and SPF-ECB forecasts as non rational or classification is not stable with respect to the forecast horizon. For SPF-ECB forecasters OLS estimation classifies both types of forecasts as non-rational for all types of loss functions.

These results clearly illustrate the fact that aggregate forecast is not consistent with the concept of rationality. So, even on the aggregate level it is difficult to admit that agents' behavior could be described as maximization of their objective function.

Section 5. Results

In addition to Diebold-Mariano test Root Mean Squared Error was computed for all models. The results of estimation for the forecast horizon 0-4 quarters ahead are reported in Table 7 - Table 9 for OLS estimation and in Table 10 - Table 12 for GMM estimation for SPF-US sample.

The results for OLS estimation are quite modest. Null hypothesis about equal forecast accuracy is not rejected for all types of loss functions and for all subsamples of forecasters. Moreover, results are the same both for mean and median.

This result could be interpreted that rationality of the forecaster does not influence the quality of the produced projection. Consequently, rationality criterion could not be seen as an acceptable way of averaging individual forecasts to obtain more precise forecast than simple average or median.

The overall situation with rationality tests based on GMM estimation is pretty much the same as with OLS estimation. However null hypothesis about equal forecast accuracy is rejected for some cases of comparing forecast accuracy of rational and non-rational subgroups of forecasters.

For forecast horizon equal two quarters ahead we rejected null hypothesis about equal forecast accuracy in favor of the alternative that rational forecasters produce more accurate projections for symmetric squared loss function both for mean and median.

For forecasting four quarters ahead according to the test rational forecasters produce more accurate projections than non-rational for asymmetric linear loss function. Results are the same both for mean and median.

However, for all cases considered above the accuracy of rational forecasters' projections equals the accuracy of the average forecast on the significance level equal to 10%.

Results for SPF-ECB forecasts for OLS and GMM estimation are provided in Table 13 and Table 14 respectively. For all definitions of the loss function, forecast horizon and method of estimation null hypothesis about equal forecast accuracy is not rejected. This means that our finding about no connection between rationality and quality of the forecast is robust to the dataset used for forecast evaluation.

Conclusion

In this paper individual forecasts from SPF-US and SPF-ECB were subject to rationality tests. Firstly, we classified all forecasters as "rational" or "non-rational" according to each type of the loss function: symmetric squared and linex for OLS estimation; symmetric squared, asymmetric squared, symmetric linear and asymmetric linear for GMM estimation. Then we calculated average forecasts (mean and median) for two subgroups and for all forecasters. After that three groups were pairwisely compared in terms of forecast's accuracy using modified Diebold-Mariano statistics. Finally, we conducted rationality test for consensus forecast for both datasets.

Our main findings could be summarized as follows:

- **Classification of the forecaster as "rational" or "non-rational" heavily relies on the assumption about loss function and employed estimation method.** The share of rational forecasters fluctuates from 30% till 60% depending on the type of loss function and dataset. GMM is more prawn to classifying forecaster as "non-rational" and OLS as "rational. Moreover, these methods sometimes give opposite results for the same type of the loss function.
- **Rationality of the forecaster could not be seen as the criterion for forecast aggregation**. For both datasets we did not find any evidence that rational forecasters make more accurate projections than non-rational ones or average forecast. That is why forecast combination based of the rationality concept does not seem to be an applicable task in spite of theoretical foundations.
- **Consensus forecasts could not be classified as "rational" for most of the cases.** GMM estimation classified forecasts as "non-rational" or results were not stable with respect to forecast horizons for both datasets. OLS technique together with linex loss function classified SPF-US consensus forecast as rational. However, the use of OLS resulted in the rejection of rationality hypothesis for SPF-ECB consensus forecast for all types of loss functions. These results suggest that individual projections could not be represented by "aggregate forecaster" whose behavior could be described as rational according to some loss function.

Based on this study several further research directions could be mentioned. For instance, it would be interesting to conduct similar research using the data for developing countries. The idea behind it is following: in US and Euro Area survey's participants are more or less homogenous. In developing countries this might not be the case. This would imply that some participants have more resources for data acquisition and processing then the others. It is equivalent to say that some experts are capable of using more information or to use data in a more efficient way then the others, resulting in the increasing chances both for the classification as "rational" and for better prediction accuracy. Under these circumstances rationality might indeed be a plausible criterion for forecast aggregation.

Another possible direction might be the exploration of the shape of the loss function employed by the forecasters as rationality test's results heavily depend on this assumption. One of the possible ways to do this might be to study documents concerning expert's objectives and employed models on the website. Another approach might be to make survey across participants. The latter approach is more costly, however, publicly available information might not be enough to draw a conclusion.

Finally, alternative approaches for rational expectations hypothesis could be proposed. For instance, past values of the economic variable could help to identify plausible interval for the future value of the variable. After identifying this interval an agent might chose her belief about future variable according some criterion such as expected loss minimization.

Appendix A. Equivalence of GMM-IV estimates and estimates from Elliott, Timmermann, Komunjer (2005)

The loss function used in Elliott, Timmermann, Komunjer (2005) has the following form:

$$L(p,\alpha,e_{t+1}) = \left(\alpha + (1-2\alpha)1_{e_{t+1}<0}\right)\left|e_{t+1}\right|^{p}.$$

Thus, its derivative can be calculated as

$$\begin{aligned} L'(p,\alpha,e_{t+1}) = p\left(\alpha + (1-2\alpha)1_{e_{t+1}<0}\right)\left|e_{t+1}\right|^{p-1}\left(-1\cdot 1_{e_{t+1}<0} + 1\cdot 1_{e_{t+1}>0}\right) = \\ = p\left((-\alpha-1+2\alpha)1_{e_{t+1}<0} + \alpha 1_{e_{t+1}>0}\right)\left|e_{t+1}\right|^{p-1} = \\ = p\left((\alpha-1)1_{e_{t+1}<0} + \alpha(1-1_{e_{t+1}<0})\right)\left|e_{t+1}\right|^{p-1} = -\,p\left(1_{e_{t+1}<0}-\alpha\right)\left|e_{t+1}\right|^{p-1}. \end{aligned}$$

As the estimate of α was obtained for fixed value of p, the derivative of the loss function is equal up to a constant $L'(p,\alpha,e_{t+1}) = \left(1_{e_{t+1}<0}-\alpha\right)\left|e_{t+1}\right|^{p-1}$.

The first order condition of the forecaster's program is $E(L'(e_{t+h})|I_t) = 0$. Then using instruments W_t and law of iterated expectations: $E([W_t L'(e_{t+h})] = E[E(W_t L'(e_{t+h})|I_t)] = E[W_t E(L'(e_{t+h})|I_t)] = 0$.

In other words, moment conditions are $E([W_t L'(e_{t+h})] = 0$. Or substituting the expression for the derivative of the loss function

$$E\left[W_t\left(1_{e_{t+1}<0}-\alpha\right)\left|e_{t+1}\right|^{p-1}\right] = 0 \quad .$$

Let us introduce matrix

$$Q_{mm} = E[m(W_t,e_{t+1},\alpha)m(W_t,e_{t+1},\alpha)'],$$

where $m(W_t,e_{t+1},\alpha)$ is moment condition. Substituting moment conditions for particular case and taking into account that W_t is a vector and all other variables are scalars the expression for Q_{mm} will be following:

$$Q_{mm} = E\left[W_t W_t'\left(1_{e_{t+1}<0}-\alpha\right)^2\left|e_{t+1}\right|^{2p-2}\right].$$

Well-known result from the GMM literature is that optimal weighting matrix for GMM-IV estimates is $\mathcal{W}^{opt} = Q_{mm}^{-1}$. Thus, GMM-IV estimate of alpha with optimal weighting matrix is the solution of the following program:

$$\begin{aligned} \hat{\alpha} = argmin\{E[m(W_t,e_{t+1},\alpha)]'\mathcal{W}^{opt}\, E[m(W_t,e_{t+1},\alpha)]\} \\ = argmin\left\{E\left[W_t\left(1_{e_{t+1}<0}-\alpha\right)\left|e_{t+1}\right|^{p-1}\right]'\mathcal{W}^{opt}\, E\left[W_t\left(1_{e_{t+1}<0}-\alpha\right)\left|e_{t+1}\right|^{p-1}\right]\right\}. \end{aligned}$$

Not to make the formulas heavy time indexes will be omitted in further calculations.

$$\frac{\partial}{\partial\alpha} = -E\left[W_t|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t(1_{e_{t+1}<0}-\alpha)|e_{t+1}|^{p-1}\right]$$
$$- E\left[W_t(1_{e_{t+1}<0}-\alpha)|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right] =$$
$$= 2\alpha E\left[W_t|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right]$$
$$- E\left[W_t|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]$$
$$- E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right] = 0$$

Taking into account that $E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right]$ is a scalar and, consequently, $E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right] =$
$= \left(E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right]\right)' =$
$\left[W_t|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]$ the estimate of α can be found as

$$\hat{\alpha} = \frac{E\left[W_t|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t(1_{e_{t+1}<0})|e_{t+1}|^{p-1}\right]}{E\left[W_t|e_{t+1}|^{p-1}\right]' \mathcal{W}^{opt}\, E\left[W_t|e_{t+1}|^{p-1}\right]}.$$

Equation (2) is nothing else as sample analogue of this equation and sample analogue for $\mathcal{W}^{opt}$ is $\hat{S}(\widehat{\alpha_T}) = T^{-1}\sum_{t=\tau}^{T+\tau-1} w_t w'_t 1_{e_{t+1}<0}|e_{t+1}|^{2p-2}$.

Appendix B. Tables

Table 7 SPF-US OLS Forecasts for horizon 0 and 1

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.230	1.266	1.230	1.220	1.266	1.220
DM	-0.067	DM	0.056	DM	0.120
p-value	0.473	p-value	0.522	p-value	0.548
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.184	1.369	1.184	1.220	1.369	1.220
DM	-0.109	DM	-0.107	DM	0.109
p-value	0.457	p-value	0.458	p-value	0.543
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.203	1.468	1.203	1.220	1.468	1.220
DM	-0.105	DM	-0.107	DM	0.106
p-value	0.458	p-value	0.458	p-value	0.542
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.203	1.468	1.203	1.220	1.468	1.220
DM	-0.106	DM	-0.106	DM	0.106
p-value	0.458	p-value	0.458	p-value	0.542
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.227	1.246	1.227	1.204	1.246	1.204
DM	-0.044	DM	0.073	DM	0.217
p-value	0.483	p-value	0.529	p-value	0.586
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.156	1.405	1.156	1.204	1.405	1.204
DM	-0.135	DM	-0.196	DM	0.125
p-value	0.447	p-value	0.422	p-value	0.550
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.175	1.482	1.175	1.204	1.482	1.204
DM	-0.106	DM	-0.113	DM	0.106
p-value	0.458	p-value	0.455	p-value	0.542
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.175	1.482	1.175	1.204	1.482	1.204
DM	-0.105	DM	-0.109	DM	0.105
p-value	0.458	p-value	0.457	p-value	0.542

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.940	1.929	1.940	1.916	1.929	1.916
DM	0.065	DM	0.094	DM	0.169
p-value	0.526	p-value	0.537	p-value	0.567
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.919	2.175	1.919	1.916	2.175	1.916
DM	-0.139	DM	0.038	DM	0.145
p-value	0.445	p-value	0.515	p-value	0.558
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.917	2.141	1.917	1.916	2.141	1.916
DM	-0.120	DM	-0.060	DM	0.121
p-value	0.452	p-value	0.476	p-value	0.548
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.917	2.141	1.917	1.916	2.141	1.916
DM	-0.126	DM	-0.079	DM	0.127
p-value	0.450	p-value	0.469	p-value	0.551
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.932	1.902	1.932	1.889	1.902	1.889
DM	0.057	DM	0.088	DM	0.132
p-value	0.523	p-value	0.535	p-value	0.552
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.890	2.193	1.890	1.889	2.193	1.889
DM	-0.129	DM	0.056	DM	0.139
p-value	0.449	p-value	0.522	p-value	0.555
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.883	2.148	1.883	1.889	2.148	1.889
DM	-0.094	DM	-0.024	DM	0.097
p-value	0.463	p-value	0.490	p-value	0.538
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.883	2.148	1.883	1.889	2.148	1.889
DM	-0.158	DM	-0.099	DM	0.160
p-value	0.437	p-value	0.461	p-value	0.564

Table 8. SPF-US OLS Forecasts for horizon 2 and 3

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.127	1.998	2.127	1.990	1.998	1.990
DM	0.108	DM	0.120	DM	0.068
p-value	0.543	p-value	0.548	p-value	0.527
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.988	2.044	1.988	1.990	2.044	1.990
DM	-0.141	DM	-0.075	DM	0.147
p-value	0.444	p-value	0.470	p-value	0.558
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.986	1.781	1.986	1.990	1.781	1.990
DM	-0.215	DM	-0.090	DM	0.224
p-value	0.415	p-value	0.464	p-value	0.588
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.986	1.781	1.986	1.990	1.781	1.990
DM	-0.237	DM	-0.091	DM	0.249
p-value	0.407	p-value	0.464	p-value	0.598
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.132	2.031	2.132	1.993	2.031	1.993
DM	0.091	DM	0.127	DM	0.137
p-value	0.536	p-value	0.550	p-value	0.554
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.996	2.044	1.996	1.993	2.044	1.993
DM	-0.126	DM	0.029	DM	0.137
p-value	0.450	p-value	0.512	p-value	0.554
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.995	1.781	1.995	1.993	1.781	1.993
DM	-0.407	DM	0.027	DM	0.456
p-value	0.342	p-value	0.511	p-value	0.675
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.995	1.781	1.995	1.993	1.781	1.993
DM	-0.491	DM	0.094	DM	0.304
p-value	0.312	p-value	0.538	p-value	0.619

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.052	2.025	2.052	1.998	2.025	1.998
DM	0.024	DM	0.077	DM	0.385
p-value	0.510	p-value	0.531	p-value	0.650
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.051	2.065	2.051	1.998	2.065	1.998
DM	-0.117	DM	0.063	DM	0.155
p-value	0.454	p-value	0.525	p-value	0.561
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.014	1.968	2.014	1.998	1.968	1.998
DM	0.014	DM	0.077	DM	0.014
p-value	0.506	p-value	0.531	p-value	0.506
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.014	1.968	2.014	1.998	1.968	1.998
DM	-0.085	DM	0.068	DM	0.122
p-value	0.466	p-value	0.527	p-value	0.549
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.053	2.014	2.053	1.991	2.014	1.991
DM	0.042	DM	0.090	DM	0.186
p-value	0.517	p-value	0.536	p-value	0.574
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.050	2.069	2.050	1.991	2.069	1.991
DM	-0.111	DM	0.036	DM	0.142
p-value	0.456	p-value	0.514	p-value	0.556
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.013	1.942	2.013	1.991	1.942	1.991
DM	0.053	DM	0.093	DM	-0.030
p-value	0.521	p-value	0.537	p-value	0.488
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.013	1.942	2.013	1.991	1.942	1.991
DM	-0.084	DM	0.100	DM	0.098
p-value	0.467	p-value	0.540	p-value	0.539

Table 9. SPF-US OLS Forecasts for horizon 4

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.146	2.123	2.146	2.107	2.123	2.107
DM	0.031	DM	0.066	DM	0.309
p-value	0.512	p-value	0.526	p-value	0.621
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.121	2.147	2.121	2.107	2.147	2.107
DM	0.069	DM	0.167	DM	0.075
p-value	0.527	p-value	0.566	p-value	0.530
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.084	2.158	2.084	2.107	2.158	2.107
DM	0.342	DM	0.177	DM	-0.015
p-value	0.634	p-value	0.570	p-value	0.494
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.084	2.158	2.084	2.107	2.158	2.107
DM	-0.064	DM	-0.059	DM	0.064
p-value	0.475	p-value	0.477	p-value	0.526
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.148	2.105	2.148	2.087	2.105	2.087
DM	0.061	DM	0.088	DM	0.284
p-value	0.524	p-value	0.535	p-value	0.612
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.103	2.073	2.103	2.087	2.073	2.087
DM	0.124	DM	0.132	DM	-0.107
p-value	0.549	p-value	0.553	p-value	0.457
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.086	2.092	2.086	2.087	2.092	2.087
DM	0.070	DM	0.053	DM	-0.066
p-value	0.528	p-value	0.521	p-value	0.474
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.086	2.092	2.086	2.087	2.092	2.087
DM	0.052	DM	-0.041	DM	-0.057
p-value	0.521	p-value	0.484	p-value	0.477

Table 10. SPF-US GMM Forecasts for horizon 0 and 1

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.268	1.245	1.268	1.220	1.245	1.220
DM	0.054	DM	0.155	DM	0.084
p-value	0.521	p-value	0.562	p-value	0.533
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.199	1.282	1.199	1.220	1.282	1.220
DM	-0.045	DM	-0.019	DM	0.077
p-value	0.482	p-value	0.492	p-value	0.531
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.287	1.212	1.287	1.220	1.212	1.220
DM	0.130	DM	0.201	DM	-0.025
p-value	0.552	p-value	0.579	p-value	0.490
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.501	1.260	1.501	1.220	1.260	1.220
DM	-0.154	DM	-0.011	DM	0.251
p-value	0.439	p-value	0.496	p-value	0.599
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.230	1.275	1.230	1.204	1.275	1.204
DM	-0.065	DM	0.128	DM	0.191
p-value	0.474	p-value	0.551	p-value	0.575
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.201	1.249	1.201	1.204	1.249	1.204
DM	-0.032	DM	-0.004	DM	0.074
p-value	0.487	p-value	0.499	p-value	0.529
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.304	1.138	1.304	1.204	1.138	1.204
DM	0.828	DM	0.282	DM	-0.164
p-value	0.795	p-value	0.611	p-value	0.435
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.470	1.245	1.470	1.204	1.245	1.204
DM	-0.103	DM	-0.107	DM	0.138
p-value	0.459	p-value	0.458	p-value	0.555

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.153	1.910	2.153	1.916	1.910	1.916
DM	0.139	DM	0.138	DM	-0.110
p-value	0.555	p-value	0.555	p-value	0.456
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.943	1.919	1.943	1.916	1.919	1.916
DM	0.277	DM	0.259	DM	0.036
p-value	0.609	p-value	0.602	p-value	0.514
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.044	1.910	2.044	1.916	1.910	1.916
DM	0.058	DM	0.083	DM	0.051
p-value	0.523	p-value	0.533	p-value	0.520
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.787	1.925	2.787	1.916	1.925	1.916
DM	-0.171	DM	0.145	DM	0.120
p-value	0.433	p-value	0.557	p-value	0.548
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.171	1.876	2.171	1.889	1.876	1.889
DM	0.162	DM	0.181	DM	-0.075
p-value	0.564	p-value	0.571	p-value	0.470
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.975	1.894	1.975	1.889	1.894	1.889
DM	0.237	DM	0.266	DM	0.102
p-value	0.593	p-value	0.605	p-value	0.540
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.073	1.881	2.073	1.889	1.881	1.889
DM	0.092	DM	0.099	DM	0.311
p-value	0.536	p-value	0.539	p-value	0.622
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.787	1.895	2.787	1.889	1.895	1.889
DM	-0.699	DM	-0.370	DM	0.107
p-value	0.245	p-value	0.357	p-value	0.543

Table 11 SPF-US GMM Forecasts for horizon 2 and 3

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.105	1.995	2.105	1.990	1.995	1.990
DM	-3.943	DM	-0.430	DM	0.364
p-value	***0.000***	p-value	0.334	p-value	0.642
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.988	2.009	1.988	1.990	2.009	1.990
DM	-0.046	DM	0.080	DM	1.652
p-value	0.482	p-value	0.532	p-value	0.949
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.067	1.989	2.067	1.990	1.989	1.990
DM	0.020	DM	0.054	DM	0.088
p-value	0.508	p-value	0.521	p-value	0.535
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	1.985	NaN	1.990	1.985	1.990
DM	NaN	DM	NaN	DM	0.119
p-value	NaN	p-value	NaN	p-value	0.547
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.088	2.007	2.088	1.993	2.007	1.993
DM	-1.493	DM	-0.398	DM	0.135
p-value	***0.070***	p-value	0.346	p-value	0.554
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.990	2.030	1.990	1.993	2.030	1.993
DM	-0.280	DM	0.043	DM	0.252
p-value	0.390	p-value	0.517	p-value	0.599
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.087	1.999	2.087	1.993	1.999	1.993
DM	-0.017	DM	0.040	DM	0.159
p-value	0.493	p-value	0.516	p-value	0.563
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	1.991	NaN	1.993	1.991	1.993
DM	NaN	DM	NaN	DM	0.170
p-value	NaN	p-value	NaN	p-value	0.567

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.171	1.996	2.171	1.998	1.996	1.998
DM	0.188	DM	0.188	DM	-0.051
p-value	0.574	p-value	0.574	p-value	0.480
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.964	1.998	1.964	1.998	1.998	1.998
DM	0.035	DM	0.036	DM	-0.013
p-value	0.514	p-value	0.514	p-value	0.495
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.028	2.000	2.028	1.998	2.000	1.998
DM	-0.184	DM	-0.185	DM	0.143
p-value	0.427	p-value	0.427	p-value	0.557
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	1.994	NaN	1.998	1.994	1.998
DM	NaN	DM	NaN	DM	0.738
p-value	NaN	p-value	NaN	p-value	0.769
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.164	1.996	2.164	1.991	1.996	1.991
DM	0.200	DM	0.206	DM	0.154
p-value	0.579	p-value	0.581	p-value	0.561
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
1.931	1.998	1.931	1.991	1.998	1.991
DM	-0.013	DM	0.006	DM	0.135
p-value	0.495	p-value	0.502	p-value	0.553
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.028	1.991	2.028	1.991	1.991	1.991
DM	-0.223	DM	-0.110	DM	0.053
p-value	0.412	p-value	0.456	p-value	0.521
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	1.991	NaN	1.991	1.991	1.991
DM	NaN	DM	NaN	DM	0.524
p-value	NaN	p-value	NaN	p-value	0.699

Table 12 SPF-US GMM Forecasts for horizon 4

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.415	2.107	2.415	2.107	2.107	2.107
DM	0.201	DM	0.196	DM	-0.023
p-value	0.579	p-value	0.577	p-value	0.491
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.170	2.115	2.170	2.107	2.115	2.107
DM	0.074	DM	0.088	DM	0.066
p-value	0.529	p-value	0.535	p-value	0.526
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.353	2.109	2.353	2.107	2.109	2.107
DM	0.145	DM	0.155	DM	0.274
p-value	0.557	p-value	0.561	p-value	0.608
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.928	2.107	2.928	2.107	2.107	2.107
DM	-1.766	DM	-1.010	DM	0.198
p-value	***0.042***	p-value	0.159	p-value	0.578
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.415	2.088	2.415	2.087	2.088	2.087
DM	0.211	DM	0.217	DM	0.028
p-value	0.583	p-value	0.586	p-value	0.511
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.162	2.090	2.162	2.087	2.090	2.087
DM	0.080	DM	0.091	DM	0.005
p-value	0.532	p-value	0.536	p-value	0.502
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.353	2.087	2.353	2.087	2.087	2.087
DM	0.177	DM	0.173	DM	-0.088
p-value	0.570	p-value	0.568	p-value	0.465
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.928	2.093	2.928	2.087	2.093	2.087
DM	-1.390	DM	-1.122	DM	0.168
p-value	***0.086***	p-value	0.134	p-value	0.567

Table 13. SPF-ECB OLS Forecasts for horizon 1 and 2 years

mean					
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.892	0.887	0.892	0.885	0.887	0.885
DM	-0.222	DM	-0.212	DM	0.239
p-value	0.413	p-value	0.417	p-value	0.594
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.888	0.890	0.888	0.885	0.890	0.885
DM	-0.257	DM	-0.243	DM	0.281
p-value	0.399	p-value	0.404	p-value	0.610
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.867	0.896	0.867	0.885	0.896	0.885
DM	-0.324	DM	-0.318	DM	0.336
p-value	0.374	p-value	0.376	p-value	0.631
median					
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.892	0.887	0.892	0.877	0.887	0.877
DM	-0.242	DM	-0.284	DM	0.233
p-value	0.405	p-value	0.389	p-value	0.592
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.890	0.888	0.890	0.877	0.888	0.877
DM	-0.241	DM	-0.256	DM	0.234
p-value	0.405	p-value	0.400	p-value	0.592
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.860	0.893	0.860	0.877	0.893	0.877
DM	-0.330	DM	-0.455	DM	0.248
p-value	0.371	p-value	0.325	p-value	0.598

mean					
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.595	2.635	2.595	2.633	2.635	2.633
DM	-0.291	DM	-0.291	DM	0.292
p-value	0.386	p-value	0.386	p-value	0.614
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.595	2.635	2.595	2.633	2.635	2.633
DM	-0.299	DM	-0.299	DM	0.301
p-value	0.383	p-value	0.383	p-value	0.618
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.590	2.635	2.590	2.633	2.635	2.633
DM	-0.394	DM	-0.361	DM	0.504
p-value	0.348	p-value	0.360	p-value	0.692
median					
linex_avmax					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.613	2.628	2.613	2.628	2.628	2.628
DM	-0.109	DM	-0.109	DM	0.322
p-value	0.457	p-value	0.457	p-value	0.626
linex_maxmin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.613	2.628	2.613	2.628	2.628	2.628
DM	-0.106	DM	-0.106	DM	0.364
p-value	0.458	p-value	0.458	p-value	0.641
linex_bin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.590	2.630	2.590	2.628	2.630	2.628
DM	-0.420	DM	-0.442	DM	0.032
p-value	0.338	p-value	0.330	p-value	0.513

Table 14. SPF-ECB GMM Forecasts for horizon 1 and 2 years

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.806	0.886	0.806	0.885	0.886	0.885
DM	0.220	DM	0.222	DM	0.156
p-value	0.587	p-value	0.587	p-value	0.562
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.855	NaN	0.855	0.885	NaN	0.885
DM	NaN	DM	0.252	DM	NaN
p-value	NaN	p-value	0.599	p-value	NaN
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	0.885	NaN	0.885	0.885	0.885
DM	NaN	DM	NaN	DM	NaN
p-value	NaN	p-value	NaN	p-value	NaN
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.858	0.853	0.858	0.885	0.853	0.885
DM	-0.613	DM	0.364	DM	0.525
p-value	0.271	p-value	0.641	p-value	0.699
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.806	0.880	0.806	0.877	0.880	0.877
DM	0.200	DM	0.211	DM	0.411
p-value	0.579	p-value	0.583	p-value	0.659
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.859	NaN	0.859	0.877	NaN	0.877
DM	NaN	DM	0.191	DM	NaN
p-value	NaN	p-value	0.575	p-value	NaN
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	0.877	NaN	0.877	0.877	0.877
DM	NaN	DM	NaN	DM	NaN
p-value	NaN	p-value	NaN	p-value	NaN
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
0.867	0.836	0.867	0.877	0.836	0.877
DM	-0.466	DM	0.299	DM	0.474
p-value	0.321	p-value	0.617	p-value	0.681

mean					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.732	2.614	2.732	2.633	2.614	2.633
DM	-0.476	DM	-0.268	DM	0.005
p-value	0.319	p-value	0.395	p-value	0.502
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	2.630	NaN	2.633	2.630	2.633
DM	NaN	DM	NaN	DM	0.243
p-value	NaN	p-value	NaN	p-value	0.595
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.732	2.614	2.732	2.633	2.614	2.633
DM	-0.502	DM	-0.229	DM	0.015
p-value	0.310	p-value	0.411	p-value	0.506
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	2.625	NaN	2.633	2.625	2.633
DM	NaN	DM	NaN	DM	-0.202
p-value	NaN	p-value	NaN	p-value	0.420
median					
symsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.732	2.599	2.732	2.628	2.599	2.628
DM	-0.740	DM	-0.301	DM	-0.277
p-value	0.233	p-value	0.383	p-value	0.391
asymsq					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	2.626	NaN	2.628	2.626	2.628
DM	NaN	DM	NaN	DM	-0.233
p-value	NaN	p-value	NaN	p-value	0.408
symlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
2.732	2.599	2.732	2.628	2.599	2.628
DM	-0.580	DM	-0.299	DM	-0.111
p-value	0.284	p-value	0.384	p-value	0.456
asymlin					
Rational	Non-Rational	Rational	All	Non-Rational	All
RMSE	RMSE	RMSE	RMSE	RMSE	RMSE
NaN	2.621	NaN	2.628	2.621	2.628
DM	NaN	DM	NaN	DM	-0.233
p-value	NaN	p-value	NaN	p-value	0.408

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